Optimal maintenance decisions under imperfect inspection

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Download the published article at
http://dx.doi.org/10.1016/j.ress.2004.10.004

Abstract

The process industry is increasingly making use of Risk Based Inspection (RBI) techniques to develop cost and/or safety optimal inspection plans. This paper proposes an adaptive Bayesian decision model to determine these optimal inspection plans under uncertain deterioration. It uses the gamma stochastic process to model the corrosion damage mechanism and Bayes' theorem to update prior knowledge over the corrosion rate with imperfect wall thickness measurements. This is very important in the process industry as current non-destructive inspection techniques are not capable of measuring the exact material thickness, nor can these inspections cover the total surface area of the component. The decision model finds a periodic inspection and replacement policy, which minimizes the expected average costs per year. The failure condition is assumed to be random and depends on uncertain operation conditions and material properties. The combined deterioration and decision model is illustrated by an example using actual plant data of a pressurized steel vessel.

Key words: Maintenance, Risk Based Inspection, gamma process, adaptive Bayesian updating, measurement error, renewal model

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Article published in Reliability Engineering and System Safety 90 (2005)

1 Introduction

In order to illustrate the requirements, which are necessary for the development of a suitable model, we start with a general introduction on current practices in the process industry concerning the use of decision models for inspection planning.

1.1 Risk Based Inspection

Since the late 1980's, numerous companies and organizations have developed several qualitative and quantitative models to aid plant engineers with the prioritization of component inspections. The average chemical process plant or refining installation will have thousands of components like pipelines, columns, heat exchangers, steam vessels etc., which operate under significant pressure. This pressure, combined with the corrosive nature of the chemicals inside the systems and the exposure to the weather on the outside, will degrade the quality of the construction material of the components. Among the most common degradation mechanisms are internal and external thinning (e.g. due to corrosion), stress corrosion cracking, brittle fracture and fatigue.

The highest uncertainties in the decision model are associated with the rate at which these mechanisms reduce the resistance of the construction material. Inspections are used to reduce this uncertainty, but since current wall thickness or crack length measurement techniques are not perfect, the measurements contain (small) errors. Some inspection techniques, e.g. ultrasonic wall thickness measurements, are highly accurate, but there is always the spatial variability in the measurements. This means that the quality of the material is not uniform for the whole component. Inspections have to be carried out such that the most critical spots are covered and such that the average measurement is representative for the complete item. Decision models should take these measurement errors into account.

1.2 Current methodologies

A common approach in tackling the inspection problem, is to start with a prioritization. This means that usually a more qualitative model is used to determine the components which constitute the highest risk. The assumption is that 80% of the risk is generated by only 20% of the components. This prioritization is then followed by a detailed quantitative analysis of these high-risk items, in which the remaining lifetimes are estimated and the consequences of failures are modelled. The American Petroleum Institute (API) has published a large document [1], which describes the methodology developed in cooperation with an industry sponsor group. This document has become a methodology in itself and is used by many companies as a basis for further development.

Typically, structural reliability methods are used to estimate the failure probabilities of the components. The condition of the construction material is described by a state function g, which is represented by a resistance (R) minus stress or load (L) model:

$$g = R - L. \tag{1}$$

The uncertain variables in this model usually have normal probability densities with suitable parameters assigned to them. The failure probability is then approximated at the limit state $g = 0$ or $R = L$ by using a reliability index method (e.g. FORM: First Order Reliability Method). Many of the methods currently applied in the process industry, also make use of a simple discrete version of Bayes' theorem to update prior knowledge on the rate of degradation with inspection results. The likelihood in Bayes' formula represents the likelihood of an inspection result correctly identifying the state of the component.

In the following sections a different approach to this problem is taken. The gamma stochastic process is used to model the degradation and the parameters of this process are updated using inspection data. This is inspired by the successful application of this stochastic process to the problem of optimally inspecting large civil structures like storm-surge barriers [2] and dikes [3]. The purpose of this paper is to illustrate the versatility of the gamma process and to show that there is great potential for its use in the process industry. Characteristically, in the development of most current models, many assumptions and simplifications are applied in order to keep the models easy to use. We will use the same assumptions in order to avoid large amounts of input and to keep the necessary input as simple as possible.

2 Modelling the deterioration

In this paper we will only consider degradation due to corrosion, but the application of this model is not restricted to this damage mechanism. For this purpose we will first define the corrosion state function, after which the gamma process with suitable parameters is introduced.

2.1 Corrosion state function

The state function for thinning due to (internal) corrosion is taken from [1] and is defined as:

$$g(t) = \underbrace{S\left(1 - \frac{Ct}{r_0}\right)}_{Resistance} - \underbrace{\frac{Pd}{2r_0}}_{Load}, \tag{2}$$

where S [MPa = 10 bar], C [mm/yr] and P [bar] are random variables representing the material strength, corrosion rate and operating pressure respectively. This model assumes that thinning will cause a failure due to ductile overload, i.e. the material becomes distorted and is no longer able to carry the stresses and loads which are present. The component diameter d [mm] and the initial material thickness r_0 [mm] are assumed to be known. The amount of wall loss at time t is given by $C \times t$ [mm]. Eq. (2) is a stress-strength model where the strength is reduced as the steel wall becomes thinner. The equivalent state function in terms of wall loss is given by

$$g(t) = \left(r_0 - \frac{Pd}{2S}\right) - Ct. \tag{3}$$

The component is assumed to fail when the limit state is reached, i.e. when $g(t) = 0$. From Eq. (3) we can deduce the critical safety margin m:

$$m = r_0 - \frac{Pd}{2S}. \tag{4}$$

This margin represents a random failure condition due to the random variables P and S.

Each component will usually have a corrosion allowance assigned to it. This is a value given by the manufacturer of the component and represents the amount of wall loss up to which the component is supposed to function safely under normal operating conditions. In general, this corrosion allowance is fairly conservative, therefore it should be less than the failure condition m. This means that we can write the corrosion allowance as a percentage of the critical safety margin: ρm with $0 \le \rho \le 1$. We will use this value as the replacement level, i.e. we will preventively replace the component if the wall loss is more than the corrosion allowance.

2.2 Gamma deterioration process

Instead of using a reliability index method, we will model the cumulative wall loss with a gamma process. We will use the following definition for the gamma density with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$:

$$\mathrm{Ga}\,(x|\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp\{-\beta x\}\, I_{(0,\infty)}(x), \tag{5}$$

where $\Gamma(\alpha) = \int_{t=0}^{\infty} t^{\alpha-1} e^{-t} dt$ is the gamma function and the indicator function is defined as: $I_{(0,\infty)}(x) = 1$ for $x \in (0,\infty)$ and zero otherwise. The stationary gamma process with shape function $at > 0$ and scale parameter $b > 0$ is a continuous-time process $\{X(t) : t \geq 0\}$ with the following properties:

(1) $X(0) = 0$ with probability one,
(2) $X(\tau) - X(t) \sim \mathrm{Ga}\,(a(\tau - t), b)$ for $\tau > t \geq 0$,
(3) $X(t)$ has independent increments.

As first proposed by Abdel-Hameed [4], let the amount of deterioration at time t be denoted by $X(t)$ and the probability density function of $X(t)$ be given by

$$f_{X(t)}(x) = \mathrm{Ga}\,(x|at, b)\,. \tag{6}$$

The amount of wall loss Ct in the state function given by Eq. (3) is therefore represented by the process $X(t)$. There are a number of advantages to using the gamma process. Most interestingly, the increments are always positive, therefore increases in wall thickness are not allowed for. The fact that increments are independent and therefore exchangeable fits well with the physics of corrosion and deterioration in general. As opposed to the reliability-index method, the gamma process properly models the temporal variability of the deterioration.

Using moment generating functions, it can be proven that the expectation and the variance of the process $X(t)$ are given by:

$$\mathbb{E}\,(X(t)) = \frac{a}{b} t \quad \text{and} \quad \mathrm{Var}\,(X(t)) = \frac{a}{b^2} t. \tag{7}$$

Assuming that the expectation and variance are linear in time, i.e.

$$\mathbb{E}(X(t)) = \mu t \quad \text{and} \quad \mathrm{Var}(X(t)) = \sigma^2 t,$$

we find that the parameters of the process $X(t)$ are defined as:

$$a = \mu^2/\sigma^2 \quad \text{and} \quad b = \mu/\sigma^2, \tag{8}$$

where μ is the average deterioration rate and σ^2 is the variance of the process. These two parameters are uncertain and assessing both variables for each individual component is very cumbersome. In order to keep the method practical and easy to use for the plant engineer, we will fix the standard deviation σ relative to the mean μ through the use of a coefficient of variation ν (this coefficient is often referred to as the COV):

$$\sigma = \nu\mu \Longrightarrow \nu = \sigma/\mu. \tag{9}$$

This approach is used in many of today's RBI models for the process industry: the coefficients of variation for the uncertain variables are predetermined by expert judgment and subsequently fixed in the model. Using (9), the probability density function for $X(t)$ in (6) reduces to

$$f_{X(t)}(x) = \text{Ga}\left(x \left| \frac{t}{\nu^2}, \frac{1}{\mu\nu^2} \right.\right). \tag{10}$$

Now only the uncertain variable μ for the average corrosion rate is left to be assessed. In the following section, a suitable prior density for this variable is defined and the consequence of fixing σ relative to μ is discussed.

3 Review of inspection models based on the gamma process

Inspection models based on the gamma process for deterioration are not new. Here we provide a short overview of the application of the models in the past. None of the following references deal with imperfect inspections, which is the topic of this paper.

Abdel-Hameed [5,6] studied an optimal periodic inspection policy model based on the class of increasing pure jump Markov processes. He provides a hypothetical example using the gamma process, which is a member of this class. In his model, failure can only be detected by inspection. Other characteristics of the model are the use of a random failure level and wear-dependent costs. Park [7] considered the same model as Abdel-Hameed, but failure is assumed to be discovered immediately without the need for an inspection. Also, Park considered a fixed failure level and the model was later generalized by Kong and Park [8] by including a random failure level. An adaptive Bayesian version of Park's model is presented by van Noortwijk et al. [9,10] in which the average rate of deterioration is uncertain. This model also includes the first application of discounted costs. Jia and Christer [11] provide essentially the same model as Park [7], but they include one more decision variable: the optimal time of the first inspection. The optimal time interval for periodic inspections then starts from the time of the first inspection. More recently, Dieulle et al. [12] and Grall et al. [13,14] treated a more extensive inspection model based on that of Park [7]. It includes an optimization for aperiodic inspections which are scheduled by means of a function of the deterioration state. Failure is detected only by inspection and a cost of unavailability of the system per unit time is included to account for repair times.

4 Inspection updating

Due to the fact that most components are usually at most inspected every 2 years, with the average inspection interval being about 6 to 8 years, there is not enough data available for a statistical analysis. Using Bayesian updating, we can efficiently incorporate the inspection measurements and the engineer's prior knowledge or estimate of the average deterioration rate μ.

4.1 Choosing the prior

Due to the large number of components and the usually limited experience of the plant engineer with probability theory, it is not feasible to ask this engineer to assess a suitable density for μ. In line with current practices in the process industry, we will only ask for an average corrosion rate over which a default density will be placed. The API [1] suggests the use of a simple discrete prior density, which is shown in Figure 1. The idea behind this density is that the

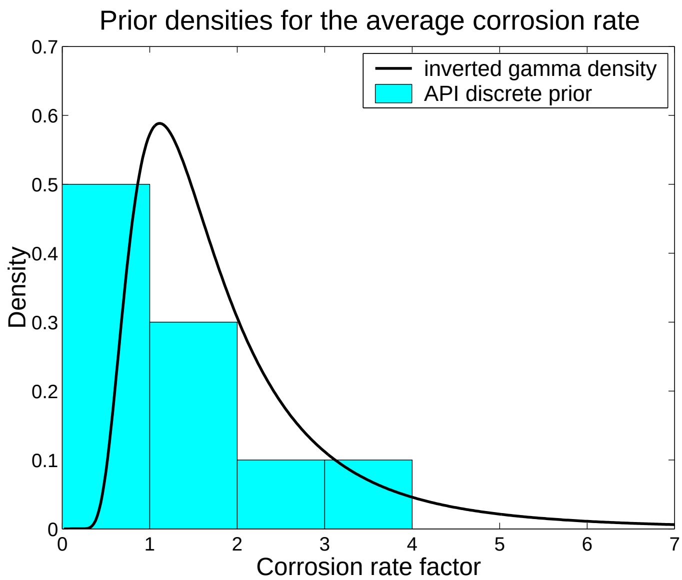

Fig. 1. The API discrete prior and the related (continuous) inverted gamma density.

model has 50% confidence in a corrosion rate which is less than or equal to the rate assessed by the plant engineer. The other 50% is divided into 30% between 1 and 2 times the assessed rate and 20% between 2 and 4 times the engineer's estimate. These probabilities are suggested when the confidence in the data or estimate is low. They are generally obtained from published data, corrosion rate tables or predefined default values [1]. The expert judgment or the generic deterioration data is not explicitly assumed to be an underestimation. We believe that engineers do not by default underestimate the corrosion rate, but prefer to be conservative and use a higher expected corrosion rate in case of uncertainty. Our experience with practices in the process industry has shown that plant engineers prefer to be 'on the safe side'. For medium and high confidence estimates, the distribution of densities for the three state categories is $\{70\%, 20\%, 10\%\}$ and $\{80\%, 15\%, 5\%\}$ respectively.

For our gamma process model we will not use this discrete prior, but a continuous inverted gamma density. The definition of this density is given by

$$\mathrm{Ig}(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \left(\frac{1}{x}\right)^{\alpha+1} \exp\left\{-\frac{\beta}{x}\right\} I_{(0,\infty)}(x). \tag{11}$$

Notice that a random variable X is inverted gamma distributed if $Y = X^{-1} \sim \mathrm{Ga}(x|\alpha,\beta)$ with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$. With suitable choices for the parameters, we can again define a default prior density for μ as is shown in Figure 1. Most confidence is placed between the engineer's assessment and 2 to 3 times this value. The prior density in Figure 1 is arbitrarily chosen for the purpose of demonstration. In practice, the parameters of this density should be determined using expert judgment. As in the discrete case, one could also define a number of different default priors from which the practitioner can select the one which best represents his own confidence/uncertainty in the degradation rate.

In the following sections we will use Bayes' theorem to update the prior density with an inspection measurement to obtain a posterior density. The continuous version of Bayes' theorem is given by

$$\pi(\mu|x) = \frac{l(x|\mu)\pi(\mu)}{\int_{\mu=0}^{\infty} l(x|\mu)\pi(\mu)d\mu}, \tag{12}$$

where $\pi(\mu)$ is the prior density, $l(x|\mu)$ the likelihood of measurement x given μ and $\pi(\mu|x)$ is the posterior density.

4.2 Case 1: perfect inspections

The choice for the inverted gamma prior becomes clear when we only consider perfect inspections. In other words, we first assume that we can measure the exact wall thickness. We can then precisely determine the amount of wall loss x at time t and it can be proven that the posterior $\pi(\mu|x)$ is given by

$$\pi(\mu|x) = \mathrm{Ig}\left(\mu \,\middle|\, \alpha + \frac{t}{\nu^2}, \beta + \frac{x}{\nu^2}\right), \tag{13}$$

where α and β are the parameters from the prior density. In Bayesian statistics, the family of inverted gamma distributions is said to be a conjugate family for samples from a gamma distribution with unknown scale parameter. This fact is also used in [9].

The equivalent of (13) for $n \geq 1$ inspections is

$$\pi(\mu|x_1,\ldots,x_n) = \mathrm{Ig}\left(\mu \,\middle|\, \alpha + \sum_{i=1}^{n} \frac{t_i - t_{i-1}}{\nu^2}\,, \beta + \sum_{i=1}^{n} \frac{x_i - x_{i-1}}{\nu^2}\right).$$

which reduces to

$$\pi(\mu|x_n) = \mathrm{Ig}\left(\mu \,\middle|\, \alpha + \frac{t_n}{\nu^2}\,, \beta + \frac{x_n}{\nu^2}\right), \tag{14}$$

if we assume that $x_0 = 0$ at $t_0 = 0$. In other words, because we fixed the standard deviation σ relative to the mean μ, only the last inspection is needed to calculate the posterior density when using perfect measurements. This is not surprising, as the last measurement supplies all the necessary information on the degradation process.

4.3 Case 2: imperfect inspections

Similar to Newby and Dagg [15], we consider a new process $Y(t)$, which includes the original process $X(t)$ together with a normally distributed measurement error ϵ:

$$Y(t) = X(t) + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon). \tag{15}$$

The same approach is also used by Whitmore [16] to include measurement error in a degradation model based on the Wiener diffusion process. In Eq. (15), we assume the error has a normal distribution with mean 0 and a standard deviation σ_ϵ. Taking a mean different from zero would mean that the inspection tends to over- or underestimate the actual wall loss. The likelihood of the measurement y given the corrosion rate μ is now determined by the convolution:

$$l(y|\mu) = f_{Y(t)}(y) = \int_{-\infty}^{\infty} f_{X(t)}(y-\epsilon) f_\epsilon(\epsilon) d\epsilon, \tag{16}$$

where $f_{X(t)}(y-\epsilon) = \mathrm{Ga}(y-\epsilon|at, b)$ is the likelihood of the gamma increment $X(t)$ as given by (10). Similarly, the likelihood for more than one inspection is given by:

$$l(\mathbf{y}|\mu) = \prod_{k=1}^{n} l_{Y(t_k)-Y(t_{k-1})}(y_k - y_{k-1}|\mu), \quad n \geq 1, \tag{17}$$

where $\mathbf{y} = (y_1, \ldots, y_n)$ are the wall loss measurements. In Eq. (17), we have used the fact that the increments are independent. Introducing some notation: the increment of $X(t)$ between two inspections is defined as $D_k = X(t_k) - X(t_{k-1})$, the difference between two measurements is $d_k = y_k - y_{k-1}$, where

$k = 1, 2, \ldots, n$. Using this notation and the integral convolution as in Eq. (16), the likelihood (17) can be rewritten as

$$l(\mathbf{y}|\mu) = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \prod_{k=1}^{n} f_{D_k}(d_k - \delta_k) f(\delta_1, \ldots, \delta_n) d\delta_1 \cdots d\delta_n, \tag{18}$$

where $\delta_k = \epsilon_k - \epsilon_{k-1}$. Clearly the δ's are not independent since every δ_k depends on δ_{k-1}. There are two options: calculate the covariances between the δ's and analytically solve the likelihood using the joint distribution of the δ's or simulate the ϵ_k's and approximate the likelihood. Since the first option will complicate matters considerably, the simulation approach is used. The likelihood (18) can be formulated as an expectation, which in turn can be approximated by the average of the product:

$$l(\mathbf{y}|\mu) = \mathbb{E}\left\{\prod_{k=1}^{n} f_{D_k}\left(d_k - \delta_k\right)\right\} \approx \frac{1}{N}\sum_{j=1}^{N}\left\{\prod_{k=1}^{n} f_{D_k}\left(d_k - \delta_k^{(j)}\right)\right\} \tag{19}$$

as $N \longrightarrow \infty$. Here the law of large numbers is applied to perform a Monte Carlo integration. For each inspection k we

(1) sample $\epsilon_k^{(j)}$ for $j = 1, 2, \ldots, N$ and
(2) calculate $\delta_k^{(j)} = \epsilon_k^{(j)} - \epsilon_{k-1}^{(j)}$, then finally
(3) calculate (19).

Note that $f_{D_k}\left(d_k - \delta_k^{(j)}\right) = 0$ when $\delta_k^{(j)} \geq d_k$ due to the definition of the gamma density in (5). Equation (19) can now be substituted in Bayes' formula (12), which can then be solved by numerical integration to obtain the posterior $\pi(\mu|y_1, \ldots, y_n)$.

Using simulation to determine the likelihood (19) greatly reduces the efficiency of the model, but the choice for the prior distribution is then no longer restricted to the inverted gamma density. Other types of distributions can be used. This feature, together with the simplified implementation, makes simulation more attractive compared to the analytical approach. Also, to assess the effect of inspection dependence, it is easy to introduce correlated measurement errors (ϵ_k) by sampling from a multivariate normal distribution. Due to the absence of the required data, this is not included here.

5 Decision criterion

The posterior probability density function for the average corrosion rate incorporates the plant engineer's prior knowledge and all the available inspection data. Using this density and the probability densities for the uncertain variables S (material strength) and P (operating pressure), the probability of failure or a preventive replacement of the component can be calculated as a function of time.

Inspections in a process plant are very expensive due to the fact that the process often has to be stopped during the inspection. Also, because many components contain highly corrosive and/or toxic chemicals, they have to be flushed and cleaned before an internal inspection can be done. The waste resulting from this rinsing needs to be recycled or treated before release, which brings added costs to the whole procedure. Therefore, the plant engineer would like to make a cost optimal decision on when to inspect the components. This means that he wants to maximize the time interval between two subsequent inspections. On the other hand, he has to ensure that the component operates safely and therefore inspections should not be executed at too large time intervals. For this purpose a cost based criterion is used, called the expected average cost per time unit; see e.g. Ross [17]. This cost criterion is derived using renewal theory and uses the concept of the component life cycle. The length of this cycle is the time from the service start until a renewal, which is either a preventive replacement or a corrective replacement due to failure.

The expected average costs per time unit are given by the ratio of the expected costs per cycle over the expected cycle length: $C(\rho, m, \Delta k) = \mathbb{E}(C_I)/\mathbb{E}(I)$, where Δk is the periodic inspection interval (i.e. the time between successive in-service inspections). The expected costs per cycle are determined by the expected number of inspections during the cycle and the expected costs due to either a preventive or a corrective replacement:

$$\mathbb{E}(C_I) = \sum_{j=1}^{\infty}(jc_I + c_P)\Pr\{X((j-1)\Delta k) \leq \rho m, \rho m < X(j\Delta k) \leq m\} + \\ [(j-1)c_I + c_F]\Pr\{X((j-1)\Delta k) \leq \rho m, X(j\Delta k) > m\}, \quad (20)$$

where c_I, c_P and c_F are resp. the costs for an inspection, a preventive replacement and a corrective replacement due to failure. The expected cycle length is given by the expected time of either a preventive replacement or failure:

$$\mathbb{E}(I) = \sum_{j=1}^{\infty} j\Delta k \Pr\{X((j-1)\Delta k) \leq \rho m, \rho m < X(j\Delta k) \leq m\} + \\ \sum_{n=(j-1)\Delta k+1}^{j\Delta k} n \Pr\{X((j-1)\Delta k) \leq \rho m, X(n-1) \leq m, X(n) > m\}. \quad (21)$$

Eqs. (20) and (21) have also been used by Park [7], van Noortwijk et al. [10] and Jia and Christer [11]. An application to the inspection and maintenance of the rock dumping of a storm-surge barrier is given by [10].

The expected average cost per time unit is a function of the replacement percentage ρ, the failure level m and the periodic inspection interval Δk. The failure level is determined by Eq. (4) and is assumed to be random due to the random variables for the operating pressure P and the material strength S. In order to include these uncertainties and the uncertainty in the deterioration rate μ for the process $X(t)$, the same technique is applied as in equation (19). We sample a large number N of sets for the three uncertain variables: $\{s^{(j)}, p^{(j)}, \mu^{(j)}\}$, for $j = 1, 2, \ldots, N$ and a Monte Carlo integration is performed to calculate $C(\rho, m, \Delta k)$ given the values of these variables. The corrosion rate can be sampled from the posterior $\pi(\mu|y_1, \ldots, y_n)$, which has been determined in section 4.

6 Case study: inspecting a pressure vessel

The gamma process decision model is illustrated with a case study on a hydrogen dryer, which is a cylindrical pressure vessel constructed from carbon steel. Table 1 summarizes the operational and measurement data, which is taken from an undisclosed plant in the Netherlands. The cost numbers are fictive. The mean material strength is determined using the tensile (s_T) and yield (s_Y) strengths with the equation:

$$S = \min\left\{1.1(s_T + s_Y)/2, s_T\right\}. \tag{22}$$

The values for these strengths can be looked up in [18] or [19]. The uncertainty in S comes from the fact that these values represent the minimum requirements for this particular material type and therefore the material could be stronger. On the other hand, the material loses strength when it is processed by the manufacturer of the component, therefore it could be weaker than indicated.

In the Netherlands, the inspection of pressurized vessels is regulated by the Dutch rules for pressure vessels [20]. All types of components are categorized and, until recently, there existed a fixed mandatory inspection interval for each category. The fixed interval for the hydrogen dryer in this example is four years, which can be clearly observed in the measurement dates. Nowadays, a plant engineer can extend the fixed interval based on the results of a RBI analysis. This is done by submitting an application to the proper authorities for acceptance. The rules require all components to have had at least one regular inspection, according to the fixed intervals, before this application can be made. For items with a fixed interval of two years, this minimum is two inspections. There will thus always be at least one set of measurement data available for the model to be applied.

Using the Bayesian updating model, which is discussed in section 4, the posterior density can be calculated for the corrosion rate given the measurement data in Table 1. In this example $\sigma_\epsilon = 0.304$ is used for all four measure-

Component and operational data:		
Component type:	vertical drum	
Material type:	carbon steel	
Service start:	1977	
Tensile strength (s_T):	413.69	MPa
Yield strength (s_Y):	206.84	MPa
Mean operating pressure (P):	32	bar(g)
Drum diameter (d):	1180	mm
Init. material thickness (r_0):	16.8	mm
Estimated corrosion rate (μ):	0.1	mm/yr
Corrosion allowance (ρm):	4.5	mm
Ultrasonic wall thickness measurements:		
1986:	15.6	mm
1990:	14.6	mm
1994:	14.2	mm
1998:	13.8	mm
Costs for different actions:		
Inspection (c_I):	10,000	\$
Preventive replacement (c_P):	50,000	\$
Failure + replacement (c_F):	1,000,000	\$

Table 1
Operational, material and inspection data for a hydrogen dryer.

ments, which corresponds to a 90% probability of the error being no more than ± 0.5mm. This is a relatively large error to account for the inaccuracy of older measurements and the fact that inspections are performed in-service and not under controlled laboratory conditions. The results are shown in Figure 2. The posterior density, which would be obtained if it were assumed that the measurements were exact, is also included for comparison purposes. In this case only the last measurement is of importance, as is determined by Eq. (14). The difference between the perfect and imperfect inspections is significant. The assumption of perfect inspections increases the confidence in the estimated corrosion rate considerably compared to the assumption of imperfect inspections. Also, the expectation for the perfect inspection posterior density is higher compared to that of the imperfect inspection posterior. This is due to the fact that the last measurement does not include the information of previous inspection measurements, which indicate a slightly slower rate of

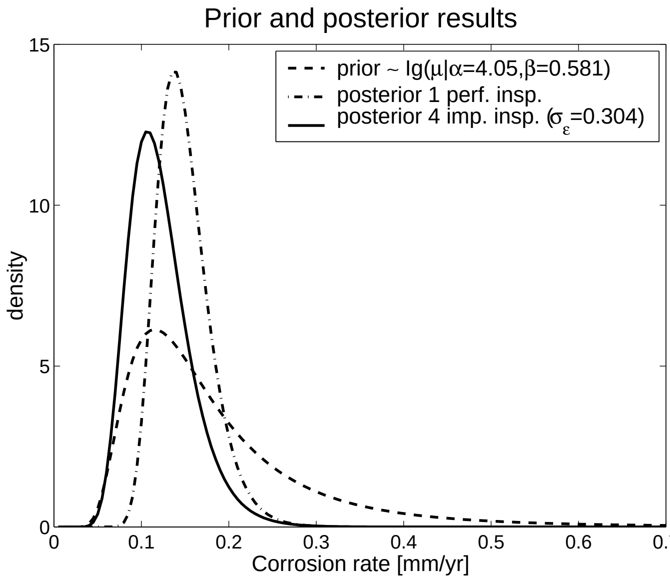

Fig. 2. Posterior densities for 1 perfect and 4 imperfect inspections compared to the prior density.

deterioration.

The coefficient of variation ν for the deterioration process is set to 1 in this example. The relationship between this coefficient of variation and the one for the random deterioration rate in classical time-dependent reliability models, e.g. the random deterioration coefficient C in Eq. (3), is given by [21].

Next, the expected average costs per year are calculated using Eqs. (20) and (21). The corrosion rate is sampled from the posterior density based on the four imperfect inspection measurements. The operating pressure P and the material strength S are sampled from normal distributions. The mean operating pressure is given in Table 1 and the standard deviation is determined by a coefficient of variation of 0.05. For the material strength, the mean is determined by Eq. (22) and the coefficient of variation is 0.20.

Figure 3 shows that the expected average costs per year will be high when inspections are performed too often. The expected annual costs also rise when the time between inspections becomes too large. The result based on the posterior density for the optimal inspection interval length is $\Delta k = 30$ years. Since the vessel was taken into service in 1977, the component's age of 27 years has to be deducted from this result. Therefore, the next inspection should be performed in 3 years from the time of evaluation, i.e. in 2007. If this inspection does not alter our assessment of the deterioration, then the following inspection should be performed 30 years later in 2037. Using the mean corrosion rate $\mathbb{E}(\mu) = 0.1212$ from the imperfect inspection posterior shown in Figure 2, the expectation of the gamma process is plotted together with the 90% confidence bounds in Figure 4. The expected time for the corrosion to reach the corro-

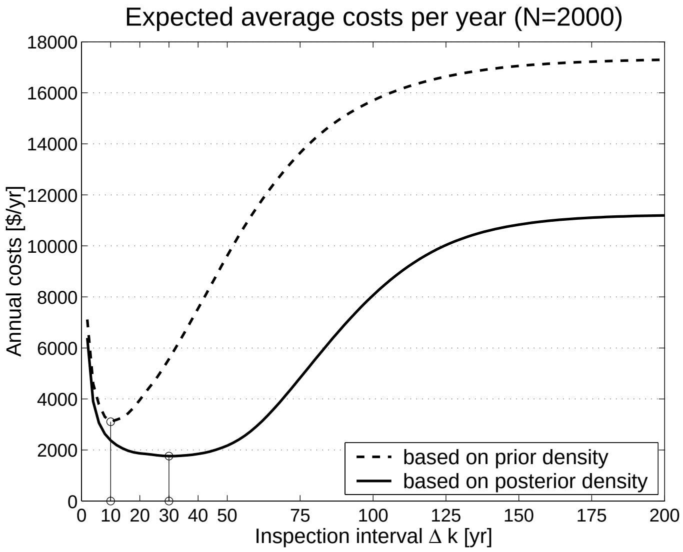

Fig. 3. Expected average costs per year for the hydrogen dryer.

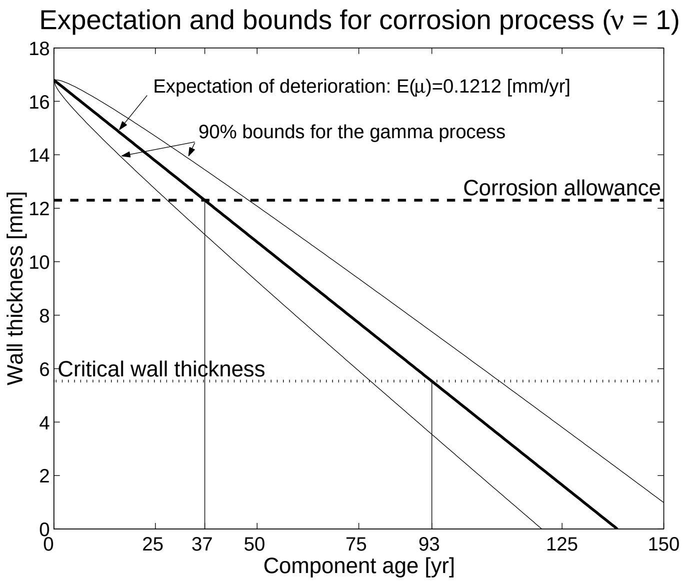

Fig. 4. Expectation and 90% confidence bounds for the gamma process using the posterior expectation of the average corrosion rate μ.

sion allowance is about 37 years. The vessel will therefore be replaced at the inspection in 2037, when it has reached an age of 60 years. This replacement will be done well before the expected time of critical failure at 93 years. A risk-averse decision maker can decide to adopt a shorter inspection interval based on the result in Figure 3. The expected annual costs for inspection interval lengths ranging from 16 to 44 years do not exceed \$2000. Choosing to perform a periodic inspection every $\Delta k = 16$ years will represent a more conservative policy without a significant increase in the expected costs per year. This policy would result in a preventive replacement of the hydrogen dryer after 48 years of service.

Finally, the result in Figure 3 for the expected average costs per year, obtained using the prior density for the average corrosion rate μ, shows that the decision model is also useful when little or no data is available. If no inspection measurements are available to the decision maker, the prior density is used to calculate the expected average costs per year. This results in a much more conservative optimal periodic inspection interval of 10 years for the example of the hydrogen dryer. Due to the higher uncertainty, the expected costs for this policy are about \$3380 per year, which is more than the expected \$1760 per year for the 30 year policy based on the posterior density. The plant manager can therefore update his information after each inspection. This approach is referred to as an adaptive Bayesian updating model; see e.g. Mazzuchi and Soyer [22,23].

Future development of this model will include the calculation of the standard deviation of the expected costs per year. Interested readers are referred to van Noortwijk [24], who presents the required equations for this purpose when costs are discounted in order to account for the time value of money.

7 Conclusions

There are a number of conclusions to be drawn from this research. First, we have observed that the gamma process is a suitable stochastic process to model the uncertain reduction of wall thickness due to corrosion. In line with what is currently done in the process industry, we have simplified the parameters of this process by fixing the ratio of the standard deviation and the mean of the deterioration rate. Together with the assumption of a fixed prior density for the average corrosion rate, this results in a model with minimum input requirements. This is a very desirable feature, as we are typically dealing with hundreds, maybe even thousands, of components in the average plant.

Next, we have created a simple extension to the Bayesian updating model, such that the model can incorporate the results from inaccurate measurements. In this step we have lost some efficiency, as we have taken the path of simulation to solve the resulting equations. However, the simulation approach allows greater flexibility in the choice of probability distributions for the variables which determine the uncertain failure level. Although the calculations for the example in this paper can be performed in a matter of minutes an a personal computer, more efforts could be made in order to effect a decrease in computation time such that multiple items can be assessed at once. These improvements can mostly be attained in the area of dedicated computer code and improved numerical integration procedures.

In order to make both cost optimal and safe inspection decisions, the cost criterion of the expected average costs per year has been used. Not only does this criterion fit well with the requirements, it also results in a graph which is easy to interpret by the plant engineers. This will ensure that the model will have some transparency and it will be less of a black box to the practitioners. Experience shows that the presentation of a single optimal value often leaves practitioners and regulators with more questions than they started out with.

The case study on a hydrogen dryer showed encouraging results of the whole model. We conclude that the use of the gamma stochastic process with an adaptive Bayesian approach to incorporating the uncertain degradation, is a viable alternative to the structural reliability methods which are commonly used in the process industry. Currently, research is being done to compare the gamma process deterioration model with the classical structural analysis models based on random variables describing the uncertain growth in deterioration [21]. The example application has also shown that the effect of imperfect inspection measurements on the outcome is considerable. The presented model takes these measurement errors into account and only requires the decision maker to supply the standard deviation of the measurement.

8 Acknowledgements

The research in this paper was partly performed by the first author during an internship at Det Norske Veritas B.V. in Rotterdam, the Netherlands. We would like to thank Chris van den Berg and his colleagues at DNV for their support and for the plant data which is used in the example.

References

[1] American Petroleum Institute, Washington D.C., Publication API581: Risk-Based Inspection - Base Resource Document, 1st Edition (May 2000).

[2] J. M. van Noortwijk, H. E. Klatter, Optimal inspection decisions for the block mats of the Eastern-Scheldt barrier, Reliability Engineering and System Safety 65 (3) (1999) 203–211.

[3] L. J. P. Speijker, J. M. van Noortwijk, M. Kok, R. M. Cooke, Optimal maintenance decisions for dikes, Probability in the Engineering and Informational Sciences 14 (1) (2000) 101–121.

[4] M. Abdel-Hameed, A gamma wear process, IEEE Transactions on Reliability 24 (2) (1975) 152–153.

[5] M. Abdel-Hameed, Inspection and maintenance policies of devices subject to deterioration, Advances in Applied Probability 19 (1987) 917–931.

[6] M. Abdel-Hameed, Correction to: “Inspection and maintenance policies of devices subject to deterioration”, Advances in Applied Probability 27 (1995) 584.

[7] K. S. Park, Optimal continuous-wear limit replacement under periodic inspections, IEEE Transactions on Reliability 37 (1) (1988) 97–102.

[8] M. B. Kong, K. S. Park, Optimal replacement of an item subject to cumulative damage under periodic inspections, Microelectronics Reliability 37 (3) (1997) 467–472.

[9] J. M. van Noortwijk, R. M. Cooke, M. Kok, A Bayesian failure model based on isotropic deterioration, European Journal of Operational Research 82 (2) (1995) 270–282.

[10] J. M. van Noortwijk, M. Kok, R. M. Cooke, Optimal maintenance decisions for the sea-bed protection of the Eastern-Scheldt barrier, in: R. Cooke, M. Mendel, H. Vrijling (Eds.), Engineering Probabilistic Design and Maintenance for Flood Protection, Kluwer Academic Publishers, Dordrecht, Netherlands, 1997, pp. 25–56.

[11] X. Jia, A. H. Christer, A prototype cost model of functional check decisons in reliability-centred maintenance, Journal of the Operational Research Society 53 (12) (2002) 1380–1384.

[12] L. Dieulle, C. Bérenguer, A. Grall, M. Roussignol, Sequential condition-based maintenance scheduling for a deteriorating system, European Journal of Operational Research 150 (2) (2003) 451–461.

[13] A. Grall, L. Dieulle, C. Bérenguer, M. Roussignol, Continuous-time predictive-maintenance scheduling for a deteriorating system, IEEE Transactions on Reliability 51 (2) (2002) 141–150.

[14] A. Grall, C. Bérenguer, L. Dieulle, A condition-based maintenance policy for stochastically deteriorating systems, Reliability Engineering and System Safety 76 (2) (2002) 167–180.

[15] M. Newby, R. Dagg, Optimal inspection policies in the presence of covariates, in: Proceedings of the European Safety and Reliability Conference – ESREL'02, Lyon, France,19-21 March, 2002, 2002, pp. 131–138.

[16] G. A. Whitmore, Estimating degradation by a Wiener diffusion process subject to measurement error, Lifetime Data Analysis 1 (1995) 307–319.

[17] S. M. Ross, Applied Probability Models with Optimization Applications, Holden-Day Inc., San Francisco, 1970.

[18] ASTM International, West Conshohocken, Philadelphia, Annual Book of ASTM Standards - Section 1: Iron and Steel Products (2003).

[19] American Society of Mechanical Engineers (ASME), New York, USA, Boiler and pressure vessel code - Section II: Materials (2001).

[20] Technical Committee on Pressure Vessels, Rules for pressure vessels, Vol. 1–3, Sdu Publishers, The Hague, the Netherlands, 1997, Dutch title: Regels toestellen onder druk.

[21] M. D. Pandey, J. M. van Noortwijk, Gamma process model for time-dependent structural reliability analysis, in: E. Watanabe, D. Frangopol, T. Utsonomiya

(Eds.), Bridge Maintenance, Safety, Management and Cost, Proceedings of the Second International Conference on Bridge Maintenance, Safety and Management (IABMAS), Kyoto, Japan, 18-22 October 2004 (CD-ROM), Taylor & Francis Group, London, 2004.

[22] T. A. Mazzuchi, R. Soyer, A Bayesian perspective on some replacement strategies, Reliability Engineering and System Safety 51 (1996) 295–303.

[23] T. A. Mazzuchi, R. Soyer, Adaptive Bayesian replacement strategies, in: J. M. Bernardo, J. O. Berger, A. P. Dawid, A. F. M. Smith (Eds.), Bayesian Statistics 5, Oxford University Press, Oxford, 1996, pp. 667–674.

[24] J. M. van Noortwijk, Explicit formulas for the variance of discounted life-cycle cost, Reliability Engineering and System Safety 80 (2) (2003) 185–195.